Computing eigenvalues of block-matrix operators in the essential spectrum

Marco Marletta
mm7@mcs.le.ac.uk
University of Leicester, United Kingdom

A block matrix operator is an operator in the form of a matrix whose entries are differential operators. In this talk we will consider a special case which has recently attracted a lot of attention in the literature, that of the block-matrix eigenvalue problem

$$\begin{pmatrix} -D^2 + p(x) & \sqrt{q(x)} \\ \sqrt{q(x)} & u(x) \end{pmatrix} \mathbf{y} = \lambda \mathbf{y}, \quad D = \frac{d}{dx},$$

on a suitable interval and subject to suitable boundary conditions. We will consider the problem of locating eigenvalues in the essential spectrum, a task which cannot be accomplished using variational methods. We will show that the problem can be solved using a type of shooting method, but that

- linear multistep methods have their order reduced to h^2 (if their usual order is > 2) or $h^2 \log(h)$ (if their usual order is 2);
- Magnus methods have their order reduced to (at best) 1, even using a singular quadrature rule;
- the order of the Magnus methods can be improved using the Niessen-Zettl transformation;
- the order of standard methods cannot be improved using the Niessen-Zettl transformation;
- in all cases, there is a method of improving the accuracy which is just as accurate as, but much cheaper than, one step of Richardson extrapolation.

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