Implementation of the Method of Variational Imbedding for Identification of Unknown Coefficient in Helmholtz Equation

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Abstract

Consider the problem of identification of the refraction index in inhomogeneous medium when the wave amplitude inside the domain D is governed by Helmholtz equation and when over-posed boundary data are available.

We "imbed" the inverse problem into a fourth-order elliptic boundary value problem for Euler-Lagrange equation being the necessary condition for minimization with respect to function u of the quadratic functional of the original equation. It is well-posed with the two boundary conditions under consideration. The Euler-Lagrange equation for $n(x, y)$ provides an explicit equation for the unknown refraction index.

The equivalence of the two problems is demonstrated and iterative procedure is devised for solving the "imbedding" boundary value problem. A featuring example is elaborated numerically.