Multi-Adaptive Galerkin Methods for ODEs

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We present the multi-adaptive Galerkin methods, mcG(q) and mdG(q), for the numerical solution of initial value problems for ODEs of the form

$$\begin{cases} \dot{u}(t) &= f(u(t),t), \ t \in (0,T], \\ u(0) &= u_0, \end{cases} \tag{1}$$

where $u : [0,T] \rightarrow \mathbb{R}^{\mathbb{N}}$, $f : \mathbb{R}^{\mathbb{N}} \times (\nvdash, \mathbb{T}] \rightarrow \mathbb{R}^{\mathbb{N}}$ is a bounded function that is Lipschitz-continuous in u, $u_0 \in \mathbb{R}^{\mathbb{N}}$ is a given initial condition and $T > 0$ a given final time.

Generalizing the standard cG(q) and dG(q) methods for ODEs, we allow in particular individual and adaptive time-steps for the different components $u_i(t)$ of the solution $u(t)$, with the objective of efficient solution of problems with several time-scales. Such problems arise in many areas of applications, such as mechanical systems, many-body problems, systems of chemical reactions and time-dependent PDEs.

We discuss the basic properties of mcG(q) and mdG(q), a priori and a posteriori error estimates, adaptive algorithms for global error control and iterative solution methods for the discrete/algebraic equations. We also describe the multi-adaptive ODE-solver *Tanganyika*, implementing mcG(q) and mdG(q) for $q \leq 20$, to achieve methods of order 40 and more.

We also present numerical results, computed with *Tanganyika*, for a variety of applications illustrating the potential of multi-adaptivity. The examples include the Lorenz system, the Solar System, chemical reaction problems, mechanical multi-scale systems, and a selection of time-dependent PDEs, such as the heat equation, the wave equation, Burger's equation and reaction-diffusion equations.