Wavelet Methods for Control Problems Involving Elliptic Boundary Value Problems: Fast Iterative Solvers

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In my talk wavelet techniques are employed for the fast numerical solution of a con- trol problem governed by an elliptic boundary value problem with boundary control. A quadratic cost functional involving natural norms of the state and the control is to be minimized. Firstly the constraint, the elliptic boundary value problem, is formulated as a saddle point problem, allowing to handle varying boundary conditions explicitly. This is combined with a ctitious domain approach in order to cover also more complicated boundaries.

Deviating from standard approaches, then (biorthogonal) wavelets are used to derive an *equivalent* infinite discretized control problem which involves only ℓ_2-norms and -operators. Classical methods from optimization yield the corresponding optimality conditions in terms of two weakly coupled (still in nite) saddle point problems for which a unique solution exists. For deriving nitedimensional systems which are uniformly invertible, stability of the discretizations has to be ensured. This together with the ℓ_2-setting circumvents the problem of *preconditioning*: all operators have *uniformly bounded* condition numbers independent of the discretization.

In order to numerically solve the resulting (finite-dimensional) linear system of the weakly coupled saddle point problems, a fully iterative method is proposed which can be viewed as an *inexact gradient* scheme. It consists of a gradient algorithm as an outer iteration which alternatingly picks the two saddle point problems, and an inner iteration to solve each of the saddle point problems, exempli ed in terms of the Uzawa algorithm. It is proved here that this strategy converges, provided that the inner systems are solved su ciently well. Moreover, since the system matrix is wellconditioned, it is shown that in combination with a *nested iteration strategy*, this iteration is asymptotically *optimal* in the sense that it provides the solution on discretization level J with an overall amount of arithmetic operations that is proportional to the number of unknowns N_J on that level.