A theory of implicit extrapolation methods for ordinary differential equations

Gennady Yu. Kulikov
kulgyu@mci.ulsu.ru
Ulyanovsk State University, Russia

At present extrapolation methods are probably the most efficient and powerful methods for solving ordinary differential equations (ODEs) numerically. They allow for the automatic determination of both stepsize and order during the computation and are easily implemented in practice.

One-step methods possessing a quadratic asymptotic expansion of the global error are of special interest since they lead to τ^2-extrapolation, where τ is the stepsize. More precisely, if the asymptotic expansion of the global error of a one-step method has only even order terms (the definition of a quadratic asymptotic expansion) then after extrapolation each row of the extrapolation table increases the order of the extrapolation method by two. This can reduce computing time dramatically.

The existing theory of symmetric one-step methods for ODEs makes it possible to identify methods whose global error has a quadratic asymptotic expansion. Most of such methods are implicit, but at present extrapolation methods are based on explicit or semi-implicit one-step methods. By this reason, we know only one quadratic extrapolation method (the GBS algorithm). Moreover, explicit form of modern extrapolation methods does not allow them to be applicable for solving stiff problems.

In the paper we construct the theory of implicit extrapolation methods and show how such methods work in practice. We also discuss problems concerning implementation of implicit methods because we cannot actually find the numerical solution from an implicit method exactly. Thus, we have to use an iterative method to solve the arising nonlinear algebraic system. But the additional error from the iterative method may influence the asymptotic global error expansion and, hence, the behavior of the extrapolation process. Numerical experiments support the theoretical results of the paper.