# Additive Runge-Kutta Schemes for Convection-Diffusion-Reaction Equations

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Additive Runge-Kutta (ARK) methods are considered for application to the spatially discretized one-dimensional convection-diffusion-reaction equation. Methods from third- to fifth-order are presented that allow for integration of stiff terms by an L-stable, stiffly-accurate explicit, singly diagonally implicit Runge-Kutta (ESDIRK) method while the non-stiff terms are integrated with a traditional explicit Runge-Kutta method (ERK). Coupling error terms are of equal order to those of the elemental methods. Derived ARK methods have vanishing stability functions for very large values of the stiff scaled eigenvalue, $z^{[I]} \rightarrow -\infty$. All constructed methods retain high stability efficiency in the absence of stiffness, $z^{[I]} \rightarrow 0$. Error control is provided by nearly L-stable embedded methods and a PID-controller. Extrapolation-type stage value predictors are provided based on dense output formulae. Methods have been optimized to minimize the leading order ARK error terms, minimize the size of the Butcher coefficients, and maximize the conservation properties. Methods have been tested on a one-dimensional convection-diffusion-reaction equation, Kap's problem, and van der Pol's equation. All methods exhibited order reduction, being most dramatic at intermediate stiffnesses.