Preconditioning for implicit Runge-Kutta methods

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A major difficulty in the implementation of implicit Runge-Kutta (IRK) methods applied to differential equations lies in the numerical solution of the underlying systems of nonlinear equations. We present a new preconditioner for their iterative solution by the inexact modified Newton method. It consists of three steps: two involve the solution of linear systems and one involves matrix-vector multiplications with the approximate Jacobian matrix. The major interest of this new preconditioner is that not only the matrix decomposition(s) can be executed in parallel, but the solution of the linear systems can be done in parallel as well. Each matrix decomposition depends on a free parameter. These parameters can be chosen to minimize for example the condition number of the preconditioned system. On serial computers when these parameters are all taken equal the decomposition cost for the preconditioner is equivalent to that of the implicit Euler method.