A Note on Symmetries Satisfied by the Ordinary Differential Equations for Population Dynamics

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In this note, we discuss symmetries or invariances satisfied by the ordinary differential equations for population dynamics such as the Lotka-Volterra equation:

$$\dot{x_i} \quad = \quad x_i(b_i - \sum_{j=1}^{n} a_{ij}x_j), \quad (i = 1, \cdots, n),$$

where x_i are time dependent variables indicating the population size of species i and a_{ij} are interaction parameters. Group theoretic properties of the equation and variants are presented. Symmetries satisfied by the population dynamics as well as those satisfied by the specific structure of the equation are discussed. As an application to stability analysis, these symmetries are used to check and constrain the form of the function $g(\{b_i\}, \{a_{ij}\})$ when stability conditions of the equilibrium points of the equation is expressed by the normalized inequality:

$$g(\{b_i\}, \{a_{ij}\}) \leq 1$$

where $g(\{b_i\}, \{a_{ij}\})$ denotes a generalized speed of population growth.