# Dynamics of Adaptive Time-Stepping ODE solvers

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For efficiency, variable time-stepping methods are often used to numerically integrate dynamical systems. The flow on chaotic attractors is often organised by the unstable manifolds of the fixed points, and it is thus necessary to obtain good numerical approximations in the neighbourhood of fixed points to reproduce the dynamics. However the standard adaptive algorithm typically fails to do this. Implicit methods designed for stiff problems are also unsuitable; they typically destroy the structure of the unstable manifold unless very small step-sizes are used. We will present examples to illustrate these poor dynamical behaviours. A stable manifold theorem for the numerical approximation of stable/unstable manifolds by fixed time-stepping one-step methods will be given, together with a limited extension to adaptive time-stepping methods. Finally we will suggest a phase space/stability based improvement to the standard algorithm.