A stabilised θ-method for circuit simulation

Stephan H.M.J. Houben
stephanh@win.tue.nl
Eindhoven University of Technology, The Netherlands

The mathematical modelling of electrical circuits leads to differential-algebraic equations (DAE) of the general form

$$\frac{d\mathbf{q}(\mathbf{q})}{dt} + \mathbf{j}(\mathbf{x}) + \mathbf{s}(t) = \mathbf{0}. \tag{1}$$

Traditionally, systems of the form 1 have been solved using Backwards Difference (BDF) methods or Implicit Runge-Kutta methods. However, such methods, in particular BDF methods, tend to damp out oscillations in the circuit. This is particular relevant when we consider the simulation of free-running oscillators. Such circuits cannot accurately be simulated with BDF methods.

The oscillations in such circuits *are* preserved by schemes like Trapezoidal Rule (TR) and also θ-methods for suitably chosen θ. Note that TR is a θ-method with $\theta = 1/2$. However, such schemes run into problems with the algebraic constraints of the DAE. They do not guarantee that the algebraic constraints are satisfied.

Our approach is based on explicitely finding the algebraic constraints of the circuit equation 1. This can be done with a topological algorithm such as described in [1]. Next, we use this information to stabilise the θ-method, i.e. we add a term which ensures that the algebraic constraints are satisfied at every time step.

The result is a one-step scheme effectively of 2nd order, which does not damp out oscillations and which correctly deals with the algebraic constraints.

References

[1] C. Tischendorf
Topological index calculation of DAEs in circuit simulation.
Surv. Math. Ind., 8 (3-4), pp. 187-199, 1999.