Waveform Relaxation Technique for Solving Forward-Backward Stochastic Differential Equations (FBSDEs)

Bevina D Handari
bevina@maths.uq.edu.au
Dept. of Mathematics, The University of Queensland, Brisbane, Australia

Forward-Backward Stochastic Differential Equations (FBSDEs) have been successfully applied to problems in mathematical finance, such as hedging of contingent claims and modelling stock sale-advertising responses. An example of a simple problem is

$$\begin{aligned} dX(t) &= \frac{X(t)}{(Z(t)-Y(t))^2+1}dt + X(t)dW(t) \\ dY(t) &= \frac{Z(t)}{(Z(t)-Y(t))^2+1}dt + Z(t)dW(t) \\ X(0) &= x; Y(T) = g(X(T)) \end{aligned}$$

whose an analytical solution is $X = Y = Z = xe^{(t/2+W(t))}, t \in [0,T]$. The problem is difficult to solve since we look for an adapted solution (X,Y,Z) where Z is still implicit in the equation. Some methods can solve this problem, including a four-step scheme where the adapted solution is obtained by solving a quasilinear partial differential equation. Instead of following that approach, we use waveform relaxation which requires us to solve problems of the form

$$dY^{k+1}(t) = h^k(t)dt + Z^{k+1}(t)dW(t)$$

at each iteration. Numerical results show improved convergence when the technique of windowing is applied along the region of integration.