Partitioned-GMRES iteration in implicit Runge-Kutta methods

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Application of the method of lines to partial differential equations leads to very large systems of ordinary differential equations, which are usually stiff. For the time integration of these systems one often relies on a low order one-stage method, or a Rosenbrock method, such that only linear systems of order N, N being the number of ODEs, have to be solved. High order, s-stage implicit Runge-Kutta methods, such as the Radau IIA methods, which are superior with respect to stability and accuracy, require the solution of (large) systems of order Ns for the internal stages, which is often considered as a serious drawback.

In this talk we will investigate the application of Krylov subspace methods to such systems. First we consider Gmres iteration applied to the complete $Ns \times Ns$ system, which involves building a Krylov subspace of $\mathbb{R}^{\mathbb{N}\sim}$ and the solution of a small least-squares problem. This approach will be expensive, as convergence will be slow as the number of stages s increases. As alternative we consider a partitioned method: Krylov subspaces of $\mathbb{R}^{\mathbb{N}}$ are formed for each of the internal stage vectors, and a slightly more complicated least-squares problem is solved to arrive at an approximate solution. We show that a reformulation of the system as a Sylvester equation allows a convenient analysis. In this partitioned Gmres-method convergence will be faster for larger values of s, and moreover for large s there is ample opportunity for parallelisation, both in the formation of the subspaces and in the construction of the orthogonal basis by the modified Gram-Schmidt process. Finally, we present numerical results of both iteration methods for several implicit Runge-Kutta methods applied to advection-diffusion equations.