Numerical Schemes for Inverse Problems of Finding Control Parameters in Diffusion Equation

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Over the last few years, it has become increasingly apparent that many physical phenomena can be described by parabolic partial differential equations with source control parameters, in one or two or three space dimensions. They arise for example, in the study of heat conduction processes, thermoelasticity, chemical diffusion and control theory, medical science. So, recent years have seen growing attention paid in the literature to the developement, analysis and implementation of accurate methods for the numerical solution of parabolic inverse problems i.e. the determination of some unknown function $p(t)$ in the parabolic partial differential equations. In this article new numerical schemes are developed for solving two parabolic inverse problems with either temperature or energy overspecification. These techniques are employed for indentifying the source control parameter $p(t)$ which satisfy $u_t = u_{xx} + u_{yy} + p(t)u + \phi$, in $R \times (0, T]$, $u(x, y, 0) = f(x, y), (x, y) \in R = [0, 1]^2$, u is known on the boundary of R and subject to the integral overspecification over a portion of the spatial domain $\int_0^1 \int_0^1 u(x, y, t)dxdy = m(t), \quad 0 \leq t \leq T$, or the overspecificatoion at a point in the spatial domain $u(x_0, y_0, t) = E(t), 0 \leq t \leq T$, where $E(t)$ is known and (x_0, y_0) is a given point of R. These schemes are considered for determining the control parameter which produces, at any given time, a desired energy distributioan in a portion of the spacial domain, or a desired temperature distribution at a given point in the spacial domain. Different finite difference techniques are used to compute the solution. These schemes have the second-order accuracy with respect to the space variables. The results of numerical experiments are presented and the accuracy and the central processor (CPU) times needed for each of the new developed methods are discussed and compared.

Key Words:
Energy Overspecification-Finite Difference Schemes, Inverse Problem, Parabolic Partial Differential Equations, Explicit Techniques, Temperature Overspecification, CPU Time, Implicit Methods, Source Control Parameters, The Order of Accuracy.