Some recent work on optimal spline collocation methods

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Piecewise polynomial collocation for the numerical solution of one-dimensional or multi-dimensional differential equations has been known for quite long time. In choosing piecewise polynomials as approximation space, one must consider the continuity of the piecewise polynomials, as an important factor that affects the efficiency of the resulting method, especially when it comes to multi-dimensional problems. There are certain advantages in using piecewise polynomials with maximum smoothness, that is, (smooth) splines, as approximating space. For example, spline collocation requires only one data (collocation) point per subinterval in one dimension (and one data point per subrectangle in two dimensions, and so on). However, it has also been known that spline collocation is sub-optimal, in the sense that it gives rise to lower order approximations than spline interpolation in the same approximation space.

Some optimal spline collocation methods have been developed for linear boundary value problems (BVPs) in one or more dimensions. In this talk, we review past work and present some recent results on optimal spline collocation methods. The recent results include the development of a sixth-order quartic spline collocation method for fourth-order BVPs and of fast solvers and preconditioners for quadratic spline collocation for second-order (multi-dimensional) BVPs.

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