Some general linear methods for stiff differential equations equations

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Consider general linear methods characterised by a partitioned $(s+r)\times(s+r)$ matrix

$$\begin{bmatrix} A & U \\ B & V \end{bmatrix}.$$

We consider methods in which the order and the stage order are each equal to p, where $r = s = p+1$ and for which A has diagonally-implicit form

$$A = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ a_{21} & \lambda & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{s1} & a_{s2} & \cdots & \lambda \end{bmatrix}.$$

Using a special condition known as "Inherent Runge-Kutta stability", methods can be derived, using only rational operations, which satisfy the order and stage-order conditions. At the same time the stability of these methods is guaranteed to be identical to that of a Runge-Kutta method. It is thus possible to find methods that are A-stable and are, at the same time, capable of inexpensive implementation.