End-effector Path Tracking for Flexible Multi-Link Planar Manipulator Through Collocation Software for Boundary-Value ODEs

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For multi-link flexible manipulators, the standard tools for solving the end-effector path tracking problem are not still efficient since, in that case, the mapping between the joint torques inputs and the end-effector position output is characterized by unstable ODEs. Then, solving the corresponding Cauchy problem definitely leads to radially unbounded joint torques. Although many ideas have been investigated -including recent works- the most interesting available results are based on iterative non-causal computations (the non-causality being necessary to bring the system ODEs to proper initial conditions leading to a stable solution for the Cauchy problem). Two principal drawbacks can be highlighted for these available solutions, namely,
1- The non-causal calculus, implying non-causal torques.
2- Some linearization (computation of the tangent linear system at some equilibrium point). This implies some restrictive constraints for those linearizing points and for the feasible trajectories. Other causal solutions have been proposed, based on input-output passivity notion defined in the control theory. However, the existence of those solutions strongly relies upon necessary assumptions on mechanical properties of the controlled manipulator.
We propose here a new formulation of the end-effector path tracking problem for the multi-link flexible manipulator. This solution is based on a nonlinear boundary-value problem formulation. We present a theoretical solution existence analysis for this boundary-value problem. Then a precise numerical solution is carried out using the collocation software *COLSYS*. The choice of a collocation method, instead of shooting techniques or other initial-value schemes, has been motivated by the unstable nature of the ODEs system under study , when formulated as initial value problem. Contrary to the presently available schemes, all of the nonlinear terms have been taken into account, without any constraints on the desired operational trajectories nor on the mechanical robot properties. Furthermore, the collocation numerical algorithm yields bounded causal solutions, without using any backward integrations. This fact, allows to overcome a well-known limitation of the above-mentioned techniques, namely, the assumption of hyperbolicity for the initial equilibrium point of the ODEs system (by hyperbolicity we mean that the tangent linear system computed at a given equilibrium point, does not admit pure imaginary eigenvalues). For this particular system,

the hyperbolicity property is insured as far as passive beams internal damping is considered. The results that we present here hold also in the absence of internal damping of the flexible links. This is very interesting, when we keep in mind the difficulty of a precise identification of these damping terms. The nice behaviour of our approach is illustrated by way of numerical examples coming from the simulation of a planar two-link flexible manipulator.