Monotonicity Preserving Multigrid Time Stepping Schemes for Linear and Nonlinear Wave Equations

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Multigrid for solving elliptic partial differential equations (PDEs) has been proven, both numerically and theoretically, to be a successful and powerful techniques. Efficient multigrid methods have also been proposed for solving non-elliptic equations, in particular, Euler and Navier Stokes equations. One approach is to accelerate the evolution of a hyperbolic system to the steady state on multiple grids by taking larger time steps on coarse grids without violating the stability (CFL) condition. Thus, the low frequency disturbances are rapidly expelled through the outer boundary whereas the high frequency errors are locally damped.

In this talk, we present two efficient multigrid time stepping schemes for scalar linear and nonlinear wave equations based on a upwind biased interpolation and restriction, and a nonstandard coarse grid update formula. Furthermore, we prove that these schemes preserve monotonicity and are total variation diminishing (TVD). Thus, no numerical oscillation is introduced, resulting in fast wave propagation to the steady state. Finally, numerical results for solving the linear wave equation and the nonlinear Burgers' equation in one and two dimensions are presented to demonstrate the effectiveness of the proposed schemes and verify that no oscillation occurs during the multigrid time stepping