Verified ODE and DAE integration controlling the wrapping effect

Martin Berz
berz@msu.edu
Michigan State University, USA

The so-called wrapping effect in verified integration algorithms is caused by the need to enclose intermediate results in rectangular boxes. It often results in a rapid growth of the overestimation on the range of final conditions that limits the predictive power of the method.

Taylor models combine high-order Taylor polynomials with interval remainder bounds and can enclose even complicated functional dependencies with little or no overestimation. Recently, Taylor model methods have been developed for the verified integration of ODE initial value problems. By propagating high-order descriptions of the flow, these methods avoid the wrapping effect to very high order and allow the verified computation of bounds on final conditions even for relatively large initial regions.

The Taylor model approach also allows a natural treatment of DAEs by recognizing the antiderivation as a natural operation on Taylor models yields a method that treats DAEs within a fully Differential Algebraic context as implicit equations made of conventional functions and the antiderivation. This method can be applied to high-index problems and allows the computation of guaranteed enclosures of final conditions from large initial regions.

We present a variety of applications of the methods, including the nonlinear Ames-Adams problem which serves as an effective nonlinear benchmark for the avoidance of the wrapping effect, the verified integration of near-earth asteroids, and the performance for a variety of DAE problems.