Linearly-implicit methods with Krylov techniques for the MOL solution of PDEs

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We consider linearly implicit one- and two-step W-methods for the MOL solution of convection-diffusion-reaction problems in 2D and 3D, where we use Krylov methods to solve the linear algebraic systems in the stages.

One-step W-(or ROW-)methods are frequently used for stiff systems. They are characterized by an easy implementation (also for variable stepsizes) and good linear stability properties (A-, L-stable). Using a special multiple Arnoldi algorithm we have constructed Krylov-W-methods, which guarantee the order of the underlying method with relatively low Krylov dimensions independent of the problem size.

Parallel two step-W-methods allow the parallel computation of the s stages if we have s processors. They are especially designed for parallel machines with few processors. The construction of methods with stage order s is possible. There are A- and L-stable methods (for constant stepsizes). Due to their high stage order there is (in contrast to one-step W-methods) in general no order reduction for very stiff problems. However, stability investigations for variable stepsizes are difficult and the methods show often a sensitive dependence of stability and error constants on stepsize changes.

The main work of the computation of the stage values can be done in parallel if Krylov methods are used for the solution of the linear systems. The speed-up is close to s for expensive right hand sides. The parallelization is done automatically in the code, a user needs to provide only the right-hand side (as usual for sequential methods).

The performance of the two types of W-methods on MOL-problems is discussed and compared with the efficient sequential code VODPK.