Multigrid Waveform Relaxation with Runge-Kutta and BVM time-discretization

Stefan G Vandewalle stefan.vandewalle@cs.kuleuven.ac.be K.U.Leuven, Dept Computerscience, Belgium

Multigrid waveform relaxation provides fast iterative methods for the solution of time dependent partial differential equations. So far, the method has been used as a PDE-solver only in combination with time discretization schemes of BDF or Crank-Nicolson type. In the current work we consider the waveform relaxation method as an iterative solver for problems with a time-discretization based on Runge-Kutta methods and on boundary value methods.

First, we consider the multigrid solution of the large linear systems that arise in implicit Runge-Kutta or Block-BVM time-stepping schemes. This is then extended towards problems in which multiple time-steps are computed simultaneously, i.e., fully discrete waveform relaxation methods or space-time grid techniques. We consider such methods for solving both isotropic as well as anisotropic and variable coefficient parabolic partial differential equations. A Fourier-Laplace analysis is used to estimate the convergence of these methods. The results of the analysis are confirmed by numerical experiments.