Mathematical Foundations of Waveform Relaxation Methods Applied to Stochastic ODEs

Henri U Schurz
schurz@math.umn.edu
University of Minnesota, School of Mathematics, USA

An operator equation

$$X \;=\; T\,X \,+\, G \tag{1}$$

in an appropriate Banach space E of adapted, cadlag stochastic processes describing an initial- or boundary value problem of a system of ordinary stochastic differential equations ((O)SDEs) is considered. Our basic assumption is that the underlying system consists of weakly coupled subsystems. Then there is some mathematical evidence that efficient waveform relaxation techniques could be used to approximate the mentioned operator equation for SDEs (as commonly seen in deterministic analysis). The proof of the convergence of corresponding waveform relaxation methods depends on the property that the spectral radius of an associated matrix is less than one. The entries of this matrix depend on the Lipschitz-constants of a decomposition of T. In proving an existence result for the operator equation we show how the entries of the matrix depend on the right hand side of stochastic differential equations. We derive conditions for the convergence under “classical” vector-valued Lipschitz-continuity of an appropriate splitting of the system of stochastic ODEs. A generalization of these key results under one-sided Lipschitz continuous and anticoercive drift coefficients of SDEs is also presented. Finally, we consider a system of SDEs with different time scales (singularly perturbed SDEs) as an illustrative example. This work presents the main results of a joint project with Klaus R. Schneider (WIAS, Berlin) running from 1997 - 2000. For the sake of convenience for the audience, we will also give a brief overview on the main iteration methods on (random) Banach spaces. Thanks to the generality of presented mathematical techniques on Banach spaces, one may be encouraged to apply the waveform relaxation methods to other types of stochastic equations.