Strong Convergence of Numerical Methods for Nonlinear Stochastic Differential Equations

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Traditional finite-time convergence theory for numerical methods applied to stochastic differential equations (SDEs) requires a global Lipschitz assumption on the drift and diffusion coefficients. In practice, many important SDE models satisfy only a local Lipschitz property and, since Brownian paths can make arbitrarily large excursions, the global Lipschitz based theory is not directly relevant. In this work we prove strong convergence results under less restrictive conditions. First, we give a convergence result for Euler-Maruyama requiring only that the SDE is locally Lipschitz and that the pth moments of the exact and numerical solution are bounded for some $p > 2$. As an application of this general theory we show that an implicit variant of Euler-Maruyama converges if the diffusion coefficient is globally Lipschitz but the drift coefficient satisfies only a one-sided Lipschitz condition; this is achieved by showing that the implicit method has bounded moments and may be viewed as an Euler-Maruyama approximation to a perturbed SDE of the same form. Second, we show that the optimal rate of convergence can be recovered if the diffusion coefficient is also assumed to behave like a polynomial. This is joint work with Xuerong Mao (Strathclyde) and Andrew Stuart (Warwick).