Adaptive Grids for High-order PDE Models (A. Vande Wouwer, P. Saucez, W.E. Schiesser and P. Zegeling)

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In this study, nonlinear wave propagation problems are investigated using an adaptive method of lines solution technique. Special attention is paid to PDE problems involving high-order spatial derivatives, e.g. several variations of the 3rd order Korteweg-de Vries (KdV) equation, the extended 4th order Fisher-Kolmogorov equation and the extended 5th order KDV equation.

In order to accurately resolve sharp spatial variations in the solution curves, an adaptive mesh refinement (AMR) algorithm based on the equidistribution principle and spatial regularization techniques is used. On the resulting highly nonuniform spatial grids, the computation of high-order derivative terms appears particularly delicate and conventional finite difference schemes (e.g. 5-point centered schemes) perform poorly. On the other hand, low-order stagewise differentiation schemes, which compute high-order derivatives by successive numerical computation of first- or second-order derivatives produce very satisfactory solutions.

These numerical tools are used to investigate existence properties of solitary wave solutions to the extended 5th order KdV equation for various values of the equation parameters. Comparisons with other numerical techniques (fixed uniform grids, nonuniform grids with predefined node movements) are presented. The use of AMR allows new solutions to be observed and investigated, where other conventional techniques perform poorly or even fail.