A framework for variational grid generation: conditioning the Jacobian matrix with matrix norms

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The first-order geometric properties of a structured grid (length of cell sides, volume of cells, corner angles, aspect ratios, grid alignment, ...) are determined by the Jacobian matrix of the induced mapping between the logical and physical regions. Once characterized the Jacobian matrices of maps inducing desired grids, global functionals can be obtained by integrating over the logical or physical domain a power of the norm of a matrix derived from the Jacobian matrix—for example by integrating the square of the Frobenius norm of the difference between the actual Jacobian matrix and the local ideal.

The approach proceeds at two levels: local (the norm of the matrix derived from the Jacobian of the induced mapping), and global(through the integration of the local functional over the whole domain).Minimizing the global functional has the effect of minimizing the local functional, unless implicit or explicit contraints are present, in which case the local functional is "equidistributed" over the domain.

When control over the "absolute" orientations of grid lines is not desired, it is advisable to use functionals which only depend on the Jacobian matrix through the corresponding metric tensor.

Many well-known grid generators fit in this framework; for example, the harmonic generator arises when one drives the metric tensor toward the identity.

Reference: A framework for variational grid generation: conditioning the Jacobian matrix with matrix norms, SIAM J. Sci. Comp. 21 (2000), no. 6, 2029-2047.