Discontinuous Galerkin Methods for Conservation Laws

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Abstract

We describe a discontinuous Galerkin (DG) method for solving hyperbolic systems of conservation laws. We review several aspects of the (DG) method including basis construction, flux evaluation, solution limiting, adaptivity, and *a posteriori* error estimation. Focusing on error estimation, we show that the leading term of the spatial discretization error of problems using piecewise-polynomial approximations of degree p is proportional to a linear combination of orthogoanl polynomials on each element of degrees p and $p+1$. These are Radau polynomials in one dimension. We also prove that the local and global discretization errors have a stonger superconvergece of order $O(h^{2(p+1)})$ and $O(h^{2p+1})$, respectively, at the outflow boundary of each element. These results are used to construct asymptotically correct *a posteriori* estimates of spatial discretization errors that are effective for linear and nonlinear conservation laws in regions where solutions are smooth.

The results of serial and parallel computations are presented for unsteady model and compressible flow problems in one, two, and three dimensions. These include mixing and other complex instabilities as well as shock problems. Solutions obtained by adaptive h- and p-refinement are compared and contrasted.