Stability analysis of Runge-Kutta methods for delay integro-differential equations

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We consider initial value problems for delay integro-differential equations of the form

$$\frac{du}{dt} = f\Big(t, u(t), u(t-\tau), \int_{t-\tau}^{t} g(t, \sigma, u(\sigma))d\sigma\Big), \quad t \geq 0,$$

$$u(t) = \varphi(t), \quad -\tau \leq t \leq 0.$$

Here, $\tau > 0$ is a constant delay, $u(t) \in R^d$, and $f,\ g,\ \varphi$ are sufficiently smooth functions. Taking a stepsize in the form $h = \tau/m$, where m is a positive integer, we can apply a Runge-Kutta method to the problem without loss of the order of the accuracy of the method. However, stability of the method, which determines a maximal size of h, is also an important factor in efficient computation.

We study stability of Runge-Kutta methods for delay integro-differential equations on the basis of the linear test equation

$$\frac{du}{dt} = Lu(t) + Mu(t-\tau) + K\int_{t-\tau}^{t} u(\sigma)d\sigma.$$

In particular, we show that the same result as in the case $K = 0$ (T. Koto, BIT **34** (1994), pp. 262–267) holds for this test equation.