Implemention issues in the numerical solution of stiff delay differential equations.

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This talk discusses the numerical solution of a general class of delay differential equations, including stiff problems, differential-algebraic delay equations, and neutral problems. The delays can be state dependent, and they are allowed to become small and vanish during the integration.

To be precise, we consider initial value problems of delay differential equations

$$\begin{aligned} M\, y'(t) &= f\Big(t, y(t), y(\alpha_1(t, y(t))), \ldots, y(\alpha_m(t, y(t)))\Big), \\ y(t_0) &= y_0, \qquad y(t) \,=\, g(t) \quad \text{for} \;\; t < t_0, \end{aligned} \tag{1}$$

where M is a constant $d \times d$ matrix and $\alpha_i(t, y(t)) \leq t$ for all $t \geq t_0$ and for all i. The value $g(t_0)$ may be different from y_0, allowing for a discontinuity at t_0.

Since we allow the matrix M to be singular, the above formulation includes all kinds of differential-algebraic delay equations. For $M = \mathrm{diag}(I, \varepsilon I)$ with a very small $\varepsilon > 0$, we get singularly perturbed problems, which form an important class of stiff problems. Moreover, neutral problems

$$y'(t) = f\Big(t, y(t), y(\alpha(t, y(t))), y'(t), y'(\alpha(t, y(t)))\Big)$$

can be written in the form (1), if we introduce a new variable $z(t) = y'(t)$ for the derivative. In fact, this problem becomes equivalent to

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y'(t) \\ z'(t) \end{pmatrix} = \begin{pmatrix} z(t) \\ z(t) - f\Big(t, y(t), y(\alpha(t, y(t))), z(t), z(\alpha(t, y(t)))\Big) \end{pmatrix}.$$

The aim of this talk is to show how implicit Runge-Kutta methods, in particular collocation methods based on Radau nodes, can be applied to solve problems of type (1).

Difficulties encountered in the implementation of implicit Runge-Kutta methods are explained, and it is shown how they can be overcome. The performances of the resulting code – RADAR5 – are illustrated on several examples.

The code, together with seven driver programs, is available at the address 'http://www.unige.ch/mat
under item 'software'.

References

[1] N. Guglielmi, E. Hairer, *Implementing Radau IIA methods for stiff delay differential equations*, Computing, to appear (2001).