Generalized polar decompositions on Lie groups

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The polar decomposition, a well known algorithm for decomposing real matrices as the product of a positive semidefinite matrix and an orthogonal matrix, is intimately related to involutive automorphisms of Lie groups and the subspace decomposition they induce. Such generalized polar decompositions, depending on the choice of the involutive automorphism σ, always exist near the identity although frequently they can be extended to larger portions of the underlying group.

In a recent paper, Munthe-Kaas, Quispel and Zanna provided an alternative proof to the local existence and uniqueness of the generalized polar decomposition by deriving differential equations obeyed by the two factors and solving them analytically, thereby providing explicit Lie-algebra recurrence relations for the coefficients of the series expansion. They also showed that when σ is a Cartan involution, the subgroup factor obeys similar optimality properties to the orthogonal polar factor in the classical matrix setting under suitable assumptions on the Lie group G.

In this talk, we discuss some properties of the generalized polar decomposition and some applications in geometric integration.