Application of Runge-Kutta methods on manifolds to solve a parabolic PDE

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Abstract

The paper presents a way of solving a parabolic PDE with Runge-Kutta methods on manifolds described in [?], [?]. These methods are designed to solve ODEs on manifolds using Lie-group theory. An essential step of Runge-Kutta methods on manifolds is a Lie algebra action which in this case corresponds to solving a simple parabolic PDE with a finite difference method or a spectral method.

This is joint work with Hans Munthe-Kaas.

Keywords: Geometric integration, differential equations on manifolds.

Summary

In [?] Hans Munthe-Kaas presents Runge-Kutta methods on manifolds which may be employed to solve many ODEs. An exponential integrator for stiff systems is discussed in that paper. This approach has led to an idea of explicit schemes for parabolic PDEs. Implicit schemes are usually employed to solve parabolic PDEs since all classical explicit schemes have a dramatic limitation on the time step in terms of the spatial discretization.

Our goal is to develop an explicit exponential type scheme which is efficient in number of flops and is not limited due to short time steps.

Many differential equations may be transformed from a Lie group to a Lie algebra. This fact is used to formulate the Runge-Kutta methods on manifolds, which solve the transformed equation in Lie algebra with 'standard' Runge-Kutta methods. A Lie algebra action is an important step of that algorithm and choosing a good Lie algebra action is a major challenge. In our case it corresponds to solving a properly chosen parabolic PDE. It may be accomplished efficiently with FFT if the boundary conditions are periodic. If the boundary conditions are non-periodic another spectral method or a finite difference method must be employed.

Results of some numerical experiments to illustrate the methods will be presented.