A low complexity Lie group method on the Stiefel manifold

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Abstract

A low complexity Lie group method for numerical integration of ordinary differential equations on the orthogonal Stiefel manifold is presented. Based on the quotient space representation of the Stiefel manifold we provide a representation of the tangent space suitable for Lie group methods. According to this representation a special type of generalized polar coordinates (GPC) is defined and used as a coordinate map. The GPC maps, recently proposed by Munthe-Kaas and Zanna, prove to adapt well to the Stiefel manifold. For the $n \times p$ matrix representation of the Stiefel manifold the arithmetic complexity of the method presented is of order np^2, and for $n \gg p$ this leads to huge savings in computation time compared to ordinary Lie group methods. Numerical experiments compare the method to a standard Lie gruop method using the matrix exponential, and conclude that on the examples presented, the methods perform equally on both accuracy and maintaining orthogonality.