Lie-group methods for highly-oscillatory ODEs

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The point of departure of this talk is an integral global-error formula, valid for all time-stepping ODE solvers. Its application to the linear oscillator $y'' + g(t)y = 0$, where $g(t) \geq 0$ and $\lim_{t\to\infty} g(t) = +\infty$, in tandem with WKB asymptotic estimates, explains why all classical methods, e.g. Runge–Kutta, deliver such poor performance. Global error can be improved a very great deal using Magnus expansions, yet even this is far from perfect. Thus, we introduce a new Lie-group solver, based on local reparametrization in a rotating frame of reference, which displays for the *Airy equation* $y'' + ty = 0$ global-error decay of $ct^{-1/4}$ (exactly like the exact solution) uniformly for all $t > 0$. In a constant-step application this means that, although the number of oscillations between grid points becomes infinite, the global error displays the correct rate of decay!