Lie group methods and DAEs

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In constructing intrinsic numerical integrators for ordinary differential equations on manifolds a common strategy is to introduce local coordinates on the manifold $\mathcal{M}$ in a neighbourhood of the point $p \in \mathcal{M}$ where the time stepping procedure is performed. Locally the integration is done with classical Runge-Kutta or multistep methods on another differential equation defined on a vector space and obtained via a change of variables.

Many possible choices are available for the coordinate mapping depending on the manifold in question. On Lie groups and homogeneous manifolds one can use Lie group actions as a tool for constructing local coordinates. On Riemannian manifolds the use of geodesics can sometimes be an option.

In the case the manifold in question is given as the null set of a map

$$F : \mathbb{R}^n \to \mathbb{R}^k$$

with no further properties specified, more general techniques based on the use of projections can be considered for constructing coordinates on the manifold.

DAEs problems after suitable reduction processes can be seen as an instance of ODEs on manifolds of the latter type.

We will describe the general idea of using local coordinates in numerical integrators on manifolds and illustrate similarities and differences of the mentioned cases.