Non-preservation of super-integrability of Hamiltonian systems by symplectic integration methods

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Among integrable 2D Hamiltonian systems, there is a subclass called the super-integrable systems in which all bounded motion are periodic and there exists an additional first integral. Examples of super-integrable systems are (i) Kepler problem, (ii) harmonic oscillator with an integer frequency ratio, and (iii) Calogero system. For these systems the additional first integral which makes the system super-integrable is not well conserved by simple symplectic integration methods and consequently the periodicity of bounded motion is not preserved. Furthermore for this first integral, it is shown that the modified first integral which is defined to be a first integral of the modified Hamiltonian does not exist. A part of present results appeared in H. Yoshida, "Non-existence of the modified first integral by symplectic integration methods", Physics Letters A, 282, 276-283 (2001).