"Slow chaos" in rigid body dynamics: analytical and numerical studies

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It is a well known fact that fast rotating rigid bodies have strong stability properties. A theoretical investigation based on that branch of Hamiltonian perturbation theory known as Nekhoroshev theory proves that this is indeed true, on very long time scales ($\sim \exp\sqrt{\omega}$, if ω is the angular velocity). However, an interesting phenomenon appears, namely, associated to the presence of low order resonances among the fast degrees of freedom, there is the possibility of "slowly chaotic" movements of the angular momentum vector in space: slowly, with speed $\sim \omega^{-1}$, the direction of the angular momentum may wander through an extended, two–dimensional region of the unit sphere, essentially filling it on relatively short time scales ($\sim \omega$). [G. Benettin and F. Fassò, *Nonlinearity* **9**, 137 (1996); G. Benettin, F. Fassò and M. Guzzo, *Nonlinearity* **10**, 1695 (1997)].

In this talk, after reviewing the problem, we describe numerical integrations which confirm the existence of this "slow chaos" in simple sample cases. [G. Benettin, A.M. Cherubini and F. Fassò, In preparation.]

Since numerical integrations have to be performed on long times, accuracy is essential and symplectic integrators are a natural choice. We describe a new approach to symplectic integration on manifolds based on switching between different systems of local coordinates (e.g. Euler angles). The algorithm uses the so called splitting method, and promises to be rather efficient. [G. Benettin, A.M. Cherubini, F. Fassò, *A "changing chart" symplectic algorithm for rigid bodies and other Hamiltonian systems on manifolds.* To appear in *SIAM Journal on Scientific Computing.*]