Lie-group methods (B. Owren)

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The systematic investigation of what is now called Lie group methods started approximately 5-6 years ago, although papers by Iserles, Lewis and Simo, as well as Crouch and Grossman dated earlier would now be included in this class of integration methods.

In the last few years, the research activity has been ever increasing, and remarkable improvements have been made regarding the mathematical foundations of the methods, their efficiency, their range of applicability, as well as their implementation. In general, one can say that Lie group methods offer a way of solving differential equations whose solution evolve on a manifold. Perhaps their most important feature is that they are intrinsic, in the sense that they are independent of any particular choice of imbedding of the manifold into a Euclidean space and a corresponding extension of the vector field ODE to a neighborhood of the manifold. In this way they differ from projection methods and many alternative ways of solving DAEs. Another attractive feature is that they facilitate a means of using a Lie group action on the manifold as a primitive as opposed to classical methods which only allow movement along straight lines. For many dynamical systems on manifolds one uses such group actions as an inherent part of the formulation of the differential equations.

Most Lie group methods involve calculation in a Lie group using local coordinates provided by its corresponding Lie algebra. A substantial part of the recent improved efficiency of Lie group methods lies in the way these coordinates are chosen, and how they are computed. The early Lie group methods of Munthe-Kaas used the exponential map from the Lie algebra to the Lie group, which by using general software involve rather expensive computations. More efficient coordinate maps have been developed and as a result, the methods we now see are at least comparable in efficiency to implicit classical integration methods, and can in some cases be even much faster, their intrinsic nature being an additional benefit.