Constraints on the spectral distribution of energy and enstrophy dissipation in forced two-dimensional turbulence

Chuong Tran
chuong@math.ualberta.ca
University of Alberta, Canada

We study two-dimensional turbulence in a doubly periodic domain driven by a spectrally localized forcing and damped by various dissipation mechanisms of the form $\nu_\eta(-\Delta)^\eta$. It is shown that for $\eta \geq 0$

$$\| u \|_1^2 \;\; \leq \;\; \lambda_{\max} \| u \|^2,$$

where $\| u \|^2$ and $\| u \|_1^2$ are the energy and enstrophy, respectively, and $\lambda_{\max}$ is an upper bound on the ratio of the enstrophy injection to the energy injection (i.e. the square of the characteristic forcing wavenumber), which is assumed to be known a priori. This result places strong constraints on the spectral distribution of energy and enstrophy and of their dissipation, and thereby on the direction of energy and enstrophy cascades. In particular, the classical dual cascade picture is shown to be invalid for forced two-dimensional Navier–Stokes turbulence ($\eta = 1$). Inclusion of Ekman drag ($\eta = 0$) along with molecular viscosity permits a dual cascade, but is incompatible with the log-modified -3 power law for the energy spectrum in the enstrophy-cascading inertial range. In order to achieve the latter, it is necessary to invoke an inverse viscosity ($\eta < 0$).