On zero mass solutions of viscous conservation laws

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In this talk, I will consider recent results from the joint work with Grzegorz Karch on the large time behavior of solutions $u = u(x,t)$ $(x \in I\!R^n,\ t > 0)$ to the Cauchy problem for the nonlinear convection-diffusion equation

$$u_t - \Delta u + a \cdot \nabla(u|u|^{q-1}) = 0, \tag{1}$$

$$u(x,0) = u_0(x). \tag{2}$$

under the assumption $u_0 \in L^1(I\!R^n)$ and $\int_{I\!R^n} u_0(x)\,dx = 0$. Here $q > 1$ and the vector $a \in I\!R^n$ are fixed.

There are many results describing the asymptotics of the solution provided that $M = \int_{I\!R^n} u_0(x)\,dx = \int_{I\!R^n} u(x,t)\,dx \neq 0$,.

In our recent work by imposing some natural additional conditions on initial data, we find the first term of the asymptotic expansion in $L^p(I\!R^n)$ of solutions to (**??**)-(**??**) with $M = 0$. In particular, we assume that u_0 satisfies $\|e^{t\Delta}u_0\|_1 \leq Ct^{-\beta/2}$ for some $\beta \in (0,1)$, all $t > 0$, and C independent of t. We show that such for a large class of initial data the solutions to the underlying heat equations have such a a decay. Under these assumptions, we also improve the known algebraic decay rates of solutions to (**??**)-(**??**) in the L^p-norms for $1 \leq p \leq \infty$. In addition, we discover *the new critical exponent*

$$q^* = 1 + \frac{1}{n+\beta}$$

such that

- for $q > q^*$ the asymptotics of solutions to (**??**)-(**??**) is

  linear and described by self-similar solutions to the heat equation;

- $q = q^*$ corresponds to the balanced case, and the asymptotics is described by a new class of self-similar solutions to equation (**??**).