Stability of a isolated fluid drop rotating with finite angular velocity

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We investigate the stability of a rotating drop of incopressible fluid held together by surface tension.

The basic flow is characterized by a shape Ω_b, and a uniform rigid rotation with angular velocity $omega_b = \omega_b e_3$, and a pressure p_b. Consideration is restricted to axisymmetric figures of equilibrium which enclose the origin. By capillarity theory it is shown that rigid rotations exist and are unique in correspondence of a finite value of angular speed ω_b, precise values can be found, e.g., in S. Chandrasekhar, Proc. R. Soc. London, A **286** 1965, R.A. Brown and L.E. Scriven, Proc. R. Soc. London, A **371** 1980.

We fix as initial data a shape Ω_0, a velocity v_0 in such a way that the angular momentum (constant) is given by

$$\int_{\Omega_0} x \times v_0 dx = \int_{\Omega_b} x \times (omega_b \times x) dx = \omega_b I_b e_3,$$

where I_b is the inertial momentum of the basic flow with respect to the x_3 axis.

In the reference frame rotating with angular velocity $omega_b$, we prove a global existence theorem for intial data “near” to the basic rest state, and we prove the exponential decay of such flow to the equilibrium configuration.

The norm employed is the Holder norm.

The result employes two technical tools: a new “a priori” estimate in a weak norm (“energy”) for the full nonlinear system of equations; an “a priori” estimate for the linearized system in a finite time interval $(t_1 - \tau, t_1)$, with constants independent of t_1.

Actually, the energy estimate is achieved by a rielaboration of generalized energy in the wake of

M. Padula, *J. Fluid Mech. and Anal.*, **1** 1998,

for the linearized problem we use the Hölder estimates obtained in

A.V. Solonnikov, Evolution free boundary value problems, *Summer course in Funchal*, 2000, to appear.

In M. Padula & V.A. Solonnikov, *Annali dell'Universita' di Ferrara*, (sez.VIII, Sci. Mat.) **46** 2000, M. Padula and A.V. Solonnikov, Quaderni di Matematica to appear,

similar results are obtained for very small angular velocity, this value is not comparable with that given in S. Chandrasekhar, Proc. R. Soc. London, A **286** 1965, R.A. Brown and L.E. Scriven, Proc. R. Soc. London, A **371** 1980. The novelty of our result lies in the choice of a different Liapunov functional that takes into account also of the conservation of angular momentum, and in the choice of the domain where linearization is made. Precisely, for the latter point we use as fixed domain Ω_b instead of the geometrically simple sphere having the same volume.

This is joint work with A.V. Solonnikov.