New Conditions for Local Regularity of a Weak Solution to the Navier–Stokes Equation

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We assume that (v, p) is a suitable weak solution of the Navier–Stokes initial value problem in $\mathbb{R}^3 \times (0, T)$. The external body force is for simplicity assumed to be zero. (x_0, t_0) is a chosen point in $\mathbb{R}^3 \times (0, T)$, whose regularity will be discussed.

We assume that $\rho > 0$ and $r > 0$ and we denote

$$\begin{aligned} \epsilon(t) &= \sqrt{t_0 - t} \quad (\text{for } t \leq t_0), \\ U_r^\rho &= \{(x,t) \in Q_T;\ t_0 - r^2/\rho^2 < t < t_0,\ \epsilon(t)\rho < |x - x_0| < r\}, \\ V_r^\rho &= \{(x,t) \in Q_T;\ t_0 - r^2/\rho^2 < t < t_0,\ |x - x_0| < \epsilon(t)\rho\}. \end{aligned}$$

We work with the conditions

$(\mathrm{A}_1)_r^\rho$ $v \in L^{a,b}(U_r^\rho)^3$ for some $a \geq 3$, $b > 3$ such that $\dfrac{2}{a} + \dfrac{3}{b} = 1$,

$(\mathrm{A}_2)_r^\rho$ $\|v\|_{L^{\infty,3}(U_r^\rho)^3} < \epsilon_1$,

$(\mathrm{B}_1)_r^\rho$ $p_- \in L^{\alpha,\beta}(V_r^\rho)$ for some $\alpha \geq 3/2$, $\beta > 3/2$ such that $\dfrac{2}{\alpha} + \dfrac{3}{\beta} = 2$,

$(\mathrm{B}_2)_r^\rho$ $\|p_-\|_{L^{\infty,3/2}(V_r^\rho)} < \epsilon_2$.

p_- denotes the negative part of pressure p: $p_- = 0$ if $p \geq 0$, $p_- = -p$ if $p < 0$. The main result is:

Theorem 1. *Suppose that there exist $\rho > 0$ and $r > 0$ such that a) condition $(\mathrm{A}_1)_r^\rho$ or condition $(\mathrm{A}_2)_r^\rho$ (with ϵ_1 sufficiently small) is satisfied and b) condition $(\mathrm{B}_1)_r^\rho$ or condition $(\mathrm{B}_2)_r^\rho$ (with ϵ_2 sufficiently small) is satisfied. Then (x_0, t_0) is a regular point of solution (v, p).*