Diffusive equilibria in granular flow

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In recent years an Enskog-type kinetic equation has been derived for inelastic collisions between granular particles. The resulting equation is expected to be a reasonable model for granular flow in rapid motion. Due to the energy losses in the interactions, the only kinetic equilibrium is a zero-temperature Maxwellian (i.e., a Dirac delta function), for which the model loses its applicability. This makes it difficult to derive corresponding macroscopic fluid equations from the moment equations, as closure relations for the moment equations usually involve knowledge of physically relevant equilibria. The situation should change when the system is immersed in a heat bath, which prevents the zero-temperature equilibria. The equation for an equilibrium is then

$$Q^e(f, f) + \epsilon \Delta_v f = 0 \tag{1}$$

where Q^e is the inelastic collieions operator (e is a parameter known as "restitution coefficient") and $\epsilon\Delta_v f$ comes from the heat bath. Solutions of (1) will therefore be called "diffusive equilibria." Equation (1) presents an unusual nonlinear and nolocal coupled integral equation/PDE problem brought to our attention by J. Carrillo.

For the case of pseudo-Maxwellian granular flow (i.e., the case where the collision kernel is independent of the relative velocity of the colliding particles) the existence of a one-parameter family of non-trivial radially symmetric solutions of (1) was recently proved by Cercignani, Illner and Stoica. The method converts (1) to an integral equation and uses the Schauder fixed point theorem.

The talk will outline this existence proof for diffusive equilibria. The main properties of the inelastic collision operator will be reviewed in the process.