Large eddy simulation and the variational multiscale method

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In many problems of practical interest, such as turbulence, it is not feasible to resolve all fine scale features numerically. Additionally, coarse scale behavior is often adequate for engineering purposes. Nevertheless, in order to accurately compute coarse scales, the effect of missing fine scales must be accounted for. In practice this amounts to employing Large Eddy Simulation (LES) and subgrid-scale models. The basic idea seems to have been originated by Smagorinsky in which an eddy viscosity is appended to the Navier-Stokes equations. The resulting system is asserted to govern the dynamics of the large eddys (i.e., coarse scales).

The classical Smagorinsky model is known to possess a number of deficiencies. Recent progress in addressing these deficiencies has been made by self-adaptive optimization of the Smagorinsky model – the so-called "dynamic model" developed at the Center for Turbulence Research at Stanford.

As an alternative for developing an improved LES model, the variational multiscale procedure is proposed. We merge a classical Smagorinsky model with a variational formulation of the Navier-Stokes equations in which the coarse and fine scales are modeled with different spaces. This setting seems to obviate many of the deficiencies of the Smagorinsky model ab initio and thereby possesses potential for improved numerical simulations. The basic concepts are described and numerical results are presented for the decay of homogeneous isotropic turbulence and equilibrium and nonequilibrium channel flows.