A Numerical Method for Shape and Stability of the Rotating Drop

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We consider the capillary problem of a drop rotating rigidly at fixed angular velocity. The centrifugal forces are balanced by surface tension alone. Such a drop is described by the equation $2\,H = 4\,r^2\,\omega^2 + C$, $\int_\Omega dx = \text{const}$. Here H denotes the mean curvature, r the orthogonal distance to the axis of rotation, C the Lagrange multiplier for the volume constraint, $\Omega \subset \mathbb{R}^3$ the region enclosed by the drop surface and ω the angular velocity.

This problem has been treated numerically by R.A. Brown and L.E. Scriven in 1980. They considered drops globally parametrised by spherical coordinates; furthermore an equatorial and a meridional reflective symmetry were imposed on the drops. By using a finite element method Brown and Scriven were able to find families of shapes branching from the axisymmetric "main" branch.

We present an algorithm which avoids an explicit global parametrisation of the drop surface and does not impose a meridional reflective symmetry (existence of the equatorial reflective symmetry has been shown by H.C. Wente in 1982).

The results of the numerical experiments performed with this algorithm extend the results formerly found by Brown and Scriven and reveal several new branches of spheroidal drop shapes. Furthermore drop shapes of toroidal type have been computed branching from an axisymmetric family of tori (the existence of the axisymmetric family was proved by R. Gulliver in 1983).