Sedimentation of Symmetric Particles in Newtonian and Viscoelastic Liquids: A Mathematical Analysis with Applications

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Over the last 40 years the study of the motion of small particles in a viscous liquid has become one of the main focuses of engineering research. The presence of the particles affects the flow of the liquid, and this, in turn, affects the motion of the particles, so that the problem of determining the flow characteristics is highly coupled. It is just this latter feature that makes any fundamental mathematical problem related to liquid-particle interaction a particularly challenging one.

Interestingly enough, even though the mathematical theory of the motion of rigid particles in a liquid is one of the oldest and most classical problems in fluid mechanics, owed to the seminal contributions of Stokes, Kirchhoff, and Thomson (Lord Kelvin) and Tait, only very recently have mathematicians become interested in a systematic study of the basic problems related to liquid-particle interaction.

This lecture concentrates on the mathematical analysis of one of the several important and still not completely understood aspects of this fascinating subject, that is, the orientation of symmetric particles sedimenting in Newtonian and viscoelastic liquids. As is well known, the general phenomenon of particle orientation plays a crucial role in many engineering problems, like the manufacturing of short-fiber composites, separation of macromolecules by electrophoresis, flow-induced microstructures, and also in blood flow problems. A first step in understanding particle orientation is to investigate particle free fall behavior (sedimentation). In this respect, it is experimentally observed, that homogeneous symmetric particles like long cylinders[1] of constant density sedimenting through a liquid will eventually orient themselves with an angle φ with the horizontal, the *tilt angle*, that depends on the physical properties of the particle (such as weight and shape) and of the liquid (such as viscosity, inertia, non-Newtonian characteristics), no matter what its initial orientation. In a purely Newtonian liquid (like water), the inertia of the liquid produces a torque on the cylinder that makes it turn with its broadside horizontal, so that the stable orientation is $\varphi = 90^0$, that thus becomes the new stable orientation. This phenomenon has a huge bear on oil industry, as will be shown by a movie. A natural guess would be that in liquids where inertia and viscoelasticity are of the same order of magnitude we should observe a tilt angle between 0^0 and 90^0, depending on how much one effect is prevailing on the other. We call this the "tilt angle phenomenon". This conjecture is substantiated by the extensive experimental work reported in on sedimentation of cylinders of different materials that do show a very stable tilt angle phenomenon with φ ranging from 0^0 to 90^0, and by the qualitative analysis performed by D.D.Joseph and J.Feng, where the tilt angle is hypothesized to arise from the balance of the inertial torque and of the viscoelastic torque generated by normal stress effects. In any case, the following issues are well-established about this phenomenon: (i) it

[1] Their length is much bigger than their diameter.

is genuinely nonlinear, (ii) it is very stable, and (iii) it is a first order effect in the Reynolds and Weissenberg numbers.[2]

One of the aims of this lecture is to show that the above conjecture about the tilt angle phenomenon, even though very plausible, is not correct, and that this phenomenon can not be attributed to the competion of inertia and normal stresses alone. Rather, other properties, like shear-thinning and/or shear thickening, must be taken into account. We shall also formulate a number open problems that deserve the attention of the interested applied mathematician and numerical analysts.

[2]Typical values are $0.01 \sim 5$ for Reynolds and $0.05 \sim 0.03$ for Weissenberg. For reference, the Reynolds number for a car moving at 30mls/hr is order of 10^4.