# Maximal Regularity of the Stokes Operator in an Infinite Cylinder

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Global weak solutions of the instationary Navier-Stokes equations pose a (famous) open problem on uniqueness and regularity in $\mathbb{R}^3$. For bounded and exterior domains $\Omega \subseteq \mathbb{R}^3$ it is known by *Leray's structure theorem* that the set of possibly singular points in time has a vanishing Haudorff measure $\mathcal{H}^{1/2}$. The essential tools in the proof of Leray's theorem are the so–called *strong energy inequality* and – in unbounded domains – the *maximal regularity* estimate of the Stokes operator $A = -P\Delta$ , i.e.,

$$\|u_t\|_{L^p(0,T;X)} + \|Au\|_{L^p(0,T;X)} \leq c\|f\|_{L^p(0,T;X)}$$

for solutions $u$ of the instationary Stokes equation $u_t + Au = f$ , $u(0) = 0$ in $X = L^q_\sigma(\Omega)$. The maximal regularity is known for $\Omega = \mathbb{R}^n$, $\mathbb{R}^n_+$, bounded and exterior domains and can e.g. be proved when the imaginary powers $A^{is}$ , $s \in \mathbb{R}$, of the Stokes operator are bounded operators in $L(X)$ .

In this talk we consider the problem of the *Stokes resolvent* $\lambda u + Au = f \in X$ and of maximal regularity in an infinite tube $\Omega = \Sigma \times \mathbb{R}$ with constant cross section $\Sigma$ . Using a partial Fourier transform w.r.t. $x_n$ the resolvent problem is transformed into an *ADN*–elliptic system with parameters $\lambda$ and the Fourier variable $\zeta \in \mathbb{R}$. This system yields a linear solution operator $M_\lambda(\zeta)$ on the space $L^r(\Sigma)$ , $1 < r < \infty$ , with a uniform estimate

$$\|M_\lambda(\zeta)\| + \|\zeta \frac{\partial M_\lambda(\zeta)}{\partial \zeta}\| \leq c.$$

Since $M_\lambda(\zeta)$ is operator-valued, the application of standard multiplier theorems is not straightforward. Therefore we refer to recent results on multiplier techniques, on $R$-bounded operator families and to improved resolvent estimates of $(R_{\lambda\zeta})$ in weighted $L^r$-spaces with arbitrary weights of Muckenhoupt type.