On the Well-Posedness of the Incompressible Euler Equations in the Besov and the Triebel-Lizorkin Spaces

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In this talk we are concerned on the initial value problem of the incompressible Euler equations in $\mathbb{R}^\varkappa$ for initial data belonging to the super-critical/critical Besov and the Triebel-Lizorkin spaces. We prove the local in time unique existence as well as the persistence of initial data regularity in these spaces. We also obtain the blow-up criterion for solutions in those spaces. In particular, in the super-critical case, our blow-up criterion results improve previous ones by Beale-Kato-Majda, Bahouri-Dehman and Kohzono-Taniuchi. As a corollary for the 2-D Euler equations, in the super-critical case, we obtain the global in time persistence of regularity characterized by these spaces. This includes Kato-Ponce's previous results for the Lebesgue(L^s_p) space as a special case. In the 2-D Euler equations with critical Triebel-Lizorkin space initial data we also prove the global in time well-posedness, the proof of which is completely different to the super-critical case. For the proof of the results we establish the Moser type of inequalities, the logarithmic inequality in the Besov and the Triebel-Lizorkin space norms of the Beale-Kato-Majda type, the commutator type of estimate, and the composition mapping estimate. The main tools are the Littlewood-Paley decomposition and the paradifferential calculus.