A numerical study of the attractor of 2D Navier-Stokes equations applied to Ocean dynamics

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The primitive equation system (PES) is one of the most general systems of partial differential equations used to study the ocean (and atmosphere) general circulation and, hence, the climate system. It has recently shown, under somewhat restrictive assumptions, the existence of the attractor for the ocean PES. Interesting enough this result, however, it is not completely satisfactory for ocean modelers since they look for having a qualitative description of such an attractor. This can be achieved by performing a large series of numerical computations in several scenarios. But the problem is that such an approach requires a great amount of computational resources, which almost irrealizable at the present stage of computing power.

Hence, one has to resort to simpler models of the modelling hierarchy established by oceanographers to study relevant features of ocean circulation. The barotropic models formulated under the assumption of the β-plane approximation are members of this hierarchy. This type of models are essentially 2D Navier-Stokes models which can be applied to simulate some significant features of the mid-latitude ocean circulation. Of course, from a physical viewpoint such models are less rich than the PES, however they still retain important properties of the ocean dynamics as its regional non-linear and dissipative characters. On the other hand, they require less computational resources to perform long term large scale computations of the kind one needs to do in order to study the properties of the attractors of mid-latitude ocean circulation.

In our presentation, we shall describe large scale numerical experiments performed with a barotropic finite element model for some scenarios of mid-latitude ocean circulation with different Reynolds numbers. To compute the main characteristics of the attractors of the circulation from the numerical output, we use the proper orthogonal decomposition (POD) technique to obtain an "optimal" orthonormal numerical basis for the manifolds that contain the attractors. Then, we do a Galerkin projection onto these subspaces to achieve low dimensional systems of differential equations which are dynamically equivalent to the fully discrete barotropic models, and finally we obtain the bifurcation diagrams of the model.

From our results, we conclude that with POD one can characterize with high accuracy stationary and periodic states of the flow using a numerical manifold of relatively low dimension, whereas it is more difficult to get a satisfactory characterization of quasi-periodic states unless one uses numerical manifolds of high dimension, say, dimension larger than O(100).