Computational Methods for Singular and Nearly Singular Integrals in Incompressible Fluid Flow

J. Thomas Beale
beale@math.duke.edu
Duke University, USA

Mathematical models of many problems in science can be formulated in terms of singular integrals. Thus there is a need for accurate and efficient numerical methods for calculating such integrals. We will describe one approach, in which we replace a singularity, or near singularity, with a regularized version, compute a sum in a standard way, and then add correction terms, which are found by asymptotic analysis near the singularity. We have used this approach to design a convergent boundary integral method for three-dimensional water waves. Boundary integral methods of this type have been used for some time; they are based on singular integrals arising from potential theory. The choice of discretization influences the numerical stability of the time-dependent method. In related work we have developed a method for computing a double layer potential on a curve, evaluated at a point near the curve. Thus values at grid points inside the curve can be found in a routine way, even for points near the boundary. This technique may offer a way to compute the influence of a moving boundary in viscous, incompressible fluid flow, since the force due to the boundary can be written in terms of single and double layer potentials.