A Large-Frequency One-Point Attractor Theory for the Incompressible Navier-Stokes Equation on Bounded Domains

Joel Avrin
jdavrin@email.uncc.edu
University of North Carolina at Charlotte, USA

We obtain a one-point attractor theory for strong solutions and for Leray solutions to the incompressible Navier-Stokes equations in three dimensions (for a variety of boundary conditions) in such a way that the forcing data can be large in the high-frequency subspaces of the Stokes operator. We do not require that the domain be thin, as in previous large-data results, but in the case of thin domains (and zero, ie. no-slip) boundary conditions) our results refine previous results obtained. Under our large-frequency assumptions we first show that trajectories of strong solutions approach each other exponentially fast in L4-norm. (That strong solutions exist for similar large-frequency assumptions on initial data as well was shown in a previous paper.)

Next, for time-independent forcing data fs, we show that if f(x,t) converges to fs in L2-norm, then the solution v for f converges to the solution w for fs in L4-norm as t goes to infinity. Under similar assumtions we show that the steady-flow equations with forcing data fs have a unique solution ws. Under the combined assumptions above we show that all (strong) solutions v converge to ws in L4-norm as t goes to infinity, thus establishing a one-point attractor for strong solutions. We then show that all Leray solutions with forcing data satisfying only a slightly more restrictive version of these conditions eventually become regular and converge to the same one-point attractor. The restrictions we invoke explicitly detail the advantages gained when either domain thinness or high-frequency assumtions are invoked, or both are invoked together.