On the hole-size bound for incomplete block designs

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An incomplete t-wise balanced design of index λ is a triple $(X, H, \mathcal{B})$ where X is a v–element set, H is a subset of X is called the hole, and $\mathcal{B}$ is a collection of subsets of X called blocks, such that every t-element subset of X is either in H or in exactly λ blocks, but not both. D.L. Kreher and R. Rees have recently shown that if H is a hole in an incomplete t-wise balanced design of order v and index λ, then $|H| \leq v/2$ if t is odd and $(v-1)/2$ if t is even. They also derived a general upper bound on the size of the hole given that its minimum block size is k: if h is the size of the hole, then

$$h \quad \leq \quad \frac{(v + (k-t)(t-2) - 1)}{(k-t+1)}.$$

We present our efforts to show that this bound is sharp infinitely often for all values of t.