Finite cyclicity of graphics of planar vector fields and Hilberts 16th problem for quadratic vector fields

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Graphics of planar vector fields can produce limit cycles under perturbation of the vector field. Whenever this is possible the phenomenon is studied by deriving the bifurcation diagram of a universal unfolding of the vector field. In the remaining cases it is sometimes possible to give bounds for the number of limit cycles appearing in a C^∞ (resp. analytic) perturbation of the vector field or at least to derive the existence of such an upper bound. This is what we call the finite cyclicity problem. We expose methods developed to derive the finite cyclicity of graphics of high codimension (or of infinite codimension inside finite-parameter analytic families). We also expose applications of the finite cyclicity problem to the finiteness part of Hilberts 16th problem for quadratic vector fields, namely the existence of a uniform bound for the number of limit cycles of a planar quadratic vector field). Indeed a program proposed by Dumortier, Roussarie and Rousseau in 1991 reduces the finiteness part of Hilberts 16th problem for quadratic vector fields to the proof that 121 graphics have finite cyclicity inside quadratic vector fields. We describe the progress in the achievement of this program.